WPI Precision Personnel Locator: Inverse Synthetic Array Reconciliation Tomography Performance

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BIOGRAPHY

Mr. Andrew Cavanaugh is a PhD. candidate in Electrical and Computer Engineering at WPI. Since completing his B.S. EE degree at The University of Rhode Island in 2008, he has served as a research assistant in the WPI Precision Personnel Location Laboratory, where he earned his M.S. degree in 2010. His research is focused on improving the accuracy of the WPI Precision Personnel Location system, using Bayesian methods to fuse diverse sources of information, and designing environmental monitoring devices for firefighters. He is a member of ION, and the IEEE.

Mr. Matthew Lowe has been attending WPI since 2005, and is working towards his PhD. degree in Electrical and Computer Engineering in the Precision Personnel Location Laboratory. Mr. Lowe has done funded research in the areas of applied mathematics, signal processing, Kalman filtering, and inertial navigation. Currently he is working on implementing a flexible framework to effectively incorporate inertial information into VSLAM applications. He is a member of ION, and the IEEE.

Dr. David Cyganski is a Professor in the Electrical and Computer Engineering department at WPI where he performs research and teaches in the areas of linear and non-linear multi-dimensional signal processing, communications and computer networks. While an active researcher in the areas of radar imaging, automatic target recognition and machine vision, he has devoted much of the past decade to developing technology for precision location, safety and situational awareness for fire-fighters within WPIs Center for First Responder Technologies. Prior to joining the faculty at WPI he was an MTS at Bell Laboratories and has held the administrative positions of Vice President of Information Systems and Vice Provost at WPI. He is a member of ION and the IEEE.

Dr. R. James Duckworth is an Associate Professor in the Electrical and Computer Engineering department at WPI. He obtained his Ph.D. in parallel processing from the University of Nottingham in England. He joined WPI in 1987. Duckworth teaches undergraduate and graduate courses in computer engineering focusing on digital and embedded system design. His main research area is developing technology for use by first responders such as location and tracking systems for indoor use. He is a fellow of the BCS, a senior member of the IEEE, and a member of ION and the IEE.

ABSTRACT

In this paper we present the theory behind a new RF-INS sensor fusion technology, as well the results of several field tests in indoor settings. The data collected from these field tests are processed with RF and INS algorithms as well as the new sensor fusion system to validate the expected performance gains.

Inverse Synthetic Aperture Array Reconciliation Tomography (ISART) is a positioning algorithm that is being developed by the WPI Precision Personnel Locator (PPL) project to track people in an indoor or GPS degraded environment. This algorithm uses inertial navigation system (INS) data to synthesize a virtual array of antennas, as is done in synthetic aperture radar (SAR) processing. This processing is applied to signals received at a set of stationary reference antennas, which, through the motion of the user, become a set of virtual antenna arrays. The received radio frequency (RF) data from these arrays are processed using the Singular Value Array Reconciliation Tomography (σART) algorithm, a novel RF positioning algorithm previously developed in support of our research program.

ISART improves upon σART by fusing sets of signals that are captured at spatially diverse locations. This spatial fusion of received signals is achieved through the tracked motion of the user who is wearing a low-cost MEMS based Inertial Measurement Unit (IMU). There is a gain in signal to noise ratio (SNR) that comes from fusing multiple data samples in time. Also, since multipath behavior tends to be non-stationary with respect to position, the ability to discount multipath contributions is enhanced by considering signals with diverse multipath profiles. It is generally accepted that inertial navigation solutions are very accurate over short periods of time, but position errors grow significantly with time. Using short segments of INS data to mitigate multipath interference can create highly accurate RF position estimates which may then be used to correct accumulated INS errors.

I. INTRODUCTION

Inverse Synthetic Array Reconciliation Tomography (ISART) is a unique algorithm that is being developed at WPI in support of the Precision Personnel Locator (PPL) project. This paper briefly describes the algorithm, but focuses on the results of three field tests that were conducted to validate the expected benefits of ISART. For a more detailed description of the algorithm itself see [1].

The ISART algorithm is a new type of sensor fusion which differs from existing RF-INS fusion systems by using INS information to mitigate multipath and noise corruption in the RF data at the raw signal level. These signals are then processed using the existing σART [2][3] algorithm, an array processing algorithm developed by the WPI PPL project specifically to address the indoor location problem with an RF-only system, under severe multipath conditions.

In order to validate the ISART algorithm, we performed three field tests in which RF and INS data were captured, and then processed using both existing RF and INS-only algorithms as well as with the ISART algorithm. The RF-only results were processed using the σART algorithm. The inertial navigation algorithms are built upon the Matlab code provided by the Open Shoe Project [4]. The Open Shoe code is also included in the ISART implementation that was used to produce the results in this paper; this removes a great deal of ambiguity when comparing the results of the ISART algorithm to that of the INS. We will only talk qualitatively about the INS results because there are a number of challenges in comparing the relative figures of merit between INS and RF tracking systems; this is exacerbated by the high degree of ambiguity concerning the initialization of the INS. These data were collected in three different field tests where each test represents a different level of operational complexity ranging from simple line of sight tests to extreme RF conditions. The size of the search area and complexity of the inertial path also increased with the more difficult RF conditions.

Before seeing the results of our experiments, it is useful to see the connections between the data, the state of the art algorithms, and the ISART algorithm. The following section will detail the workings of the ISART algorithm with only the relevant aspects of the σART and INS algorithms. For more information on foot-mounted INS algorithms the Open Shoe [4] and NavShoe [5] papers are excellent resources. The σART algorithm will be briefly explained in the following section, but for more details see [6] and [2].

II. ISART THEORY

A. PPL System Description

The PPL system is designed to locate first responders indoors with sub meter positioning accuracy. The ISART algorithm tracks users wearing an RF transmitter and an IMU. The RF signals are received by stationary reference antennas on the exterior of the building, and the IMU data is sent over an RF data channel that operates in a separate frequency band. The PPL system is designed to be deployed rapidly, and configure itself in an ad-hoc mode, requiring no preinstalled infrastructure.. The ISART algorithm is designed to reject non-idealities introduced by hardware and mitigate the deleterious effects of multipath on the positioning accuracy of the system.

Both the mobile transmitters and stationary receivers are software defined radios that can be programmed to transmit or receive user-defined multicarrier-wideband (MCWB) signals in the 550-700 MHz. band. These fully custom units use FPGAs to interface between the user and the RF hardware chain. The received signals on the receivers are fed via ethernet (wired or wireless) to a base station computer, where the positioning algorithms are implemented in Matlab, and the data can be stored for future post-processing.

The transmitter is not synchronized to the receivers, so the received data does not contain time of arrival (TOA) information, but the synchronization of the receivers does encode time-difference of arrival (TDOA) information into the received signal. One could compute TDOAs between each pair of reference antennas and compute a positing solution, but the ISART algorithm uses a fundamentally different approach. Unlike existing TOA/TDOA algorithms, the ISART algorithm considers the entire set of received signals and exhaustively scans over a search space, evaluating a metric, to determine position estimate.

Multicarrier Wide Band (MCWB) signal structure: The MCWB signal structure is used in the navigation signal transmitted by the mobile device. This signal consists of a sum of unmodulated sinusoids, evenly spaced in frequency. The use of unmodulated carriers allows the signal to occupy very little bandwidth, and fit between existing services. The current implementation of the PPL hardware transmits in the 550-700 MHz. band. Control signals and data are transmitted over a separate radio operating at 915 MHz. The frequency domain representation of a general MCWB signal is given by Equation (1):

$$X(\omega) = \sum_{n=0}^{N-1} \delta(\omega - (\omega_0 - n\Delta\omega)) \tag{1}$$

Where N is the total number of carriers (usually $\approx$ 100), ω_0 is the lowest frequency, and $\Delta\omega$ is the spacing between adjacent carrier frequencies. The quantities $\Delta\omega$ and ω_0 are chosen in such a way as to align the carrier frequencies with Discrete Fourier Transform (DFT) frequencies, this way we need only one DFT coefficient per carrier. Practical considerations force us to avoid certain regions of spectrum. For example, the region between 608 and 612 MHz., which is reserved for emergency services. This does not fundamentally change the analysis moving forward, as we simply mask out these frequency bins. This masking also allows us to avoid TV stations and other interferers which can be identified during the system configuration phase.

Without loss of generality we can assume that the carriers in Equation (1) have no initial phase. In reality the phase is given to reduce the signal's crest factor in accordance with the results of Boyd [7], in applying the work of Newman [8] to the crest factor problem. Since the initial phase is known, it can be calibrated out of the received signal, and does not need to be considered for the analysis of this section. The crest factor reduction is an important step in any hardware implementation as it mitigates the problem of dynamic range compression when sampling our signals.

The MCWB signal is unmodulated, and the transmitter can be thought of as 'always on' in the sense that we are not rapidly 'pulsing' our signal to attempt to achieve finer time resolution (this would, of course, distort the signal and produce out-of-band emissions). When this is coupled with the fact that the transmitter is not synchronized with the receivers we encounter the problem of aliasing. A sum of sinusoids with $\Delta\omega \in \mathbb{Q}$ will have a period determined by the reciprocal of the greatest common multiple among the individual frequencies [6]. In practice we need to construct our signal such that the aliasing period corresponds to a range that is larger than the propsed area of operations. We use a ≈ 200 m aliasing window in all of the experiments conducted for this paper. The period of a MCWB signal is given in Equation (2)

$$T_{\text{alias}} = \frac{1}{2\pi\Delta\omega} = \frac{1}{\Delta f} \tag{2}$$

Multiplying this period by c_0 yields the aliasing window in range (meters).

Asynchronous mobile transmitter: The ideal transmitted signal shown in Equation (1) is altered by the mismatches between the transmitter's and receiver's sample clock and RF mixer frequencies. Amendolare [6] and Breen [9] have performed analyses, based on the local oscillators (LO) employed in the PPL system, and have concluded that the LO frequency mismatch between the transmitter and receiver are small enough that we need only to consider the effects of sample clock offsets and the differences in mixer frequencies. The local oscillators in the transmitters and receivers provide the frequency reference for both the DAC/ADC as well as the RF mixer. The net effects of having different oscillators are the introduction of an unknown time offset, $\tilde{\tau}(t)$, from the sample clock offset, and an unknown phase offset, $\tilde{\theta}(t)$, from the mixer frequency offset. These two parameters can be assumed constant over the time frame of a data capture. This leaves us with a time offset $\tilde{\tau}$, and a frequency dependent phase offset $\tilde{\theta}$. The received signal becomes:

$$X'(\omega) = X(\omega)H(\omega)e^{-j(\omega\tilde{\tau}-\tilde{\theta})} \tag{3}$$

where $H(\omega)$ is the channel response between the transmit and receive antennas, and $X(\omega)$ is the ideal transmitted signal from Equation (1).

Received data matrix: As previously mentioned, the σART algorithm operates on the set of received signals, rather than using individual signals or pairs of signals to compute ranges or TDOAs. The data captured at each reference antenna are stored as columns of a received data matrix, $\mathbf{R} \in \mathbb{C}_{N\times p}$. Each row of $\mathbf{R}$ corresponds to the DFT bin of a carrier, and each column corresponds to a reference antenna. The number of rows and number of columns, N and P, represent the total number of carriers and the total number of reference antennas. The received signal from the p^{th} antenna is given by Equation (5):

$$\mathbf{r}_{p,\text{ideal}} = X(\omega)H_p(\omega) \tag{4}$$

If we consider the non-ideal time and phase offsets, this becomes:

$$\mathbf{r}_p = X(\omega)H_p(\omega)e^{-j(\omega\tilde{\tau}_p-\tilde{\theta}_p)} \tag{5}$$

In matrix form, we can write the modified matrix as Equation (6)

$$\mathbf{R} = \begin{pmatrix} e^{-j\omega\tilde{\tau}_1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & e^{-j\omega\tilde{\tau}_p} \end{pmatrix} \breve{\mathbf{R}} \begin{pmatrix} e^{j\tilde{\theta}_1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & e^{j\tilde{\theta}_p} \end{pmatrix} \tag{6}$$

where $\breve{\mathbf{R}} = X(\omega)[H_1(\omega)\dots H_p(\omega)]$. It is important to note that these time and phase shifts are unitary operators and do not effect the energy of the received data matrix [6].

B. Singular Value Array Reconciliation Tomography

Singular Value Array Reconciliation Tomography (σART) is an RF localization algorithm which uses the received signals from a set of stationary reference antennas to locate a mobile transmitter. The inputs to this algorithm are a set of captured RF data, a discretized representation of the area being searched, and corresponding reference antenna coordinates.

Given a received data matrix, $\mathbf{R}$, a set of reference coordinates, and a discretized representation of the area to be searched, the σART algorithm will return a position estimate, and a metric function which corresponds to the likelihood that the locator is at a given position [10].

1) Spatial scanning: Given a discretized search space, or scangrid, the σART algorithm computes a metric at each point in the scangrid and returns the location at which the metric is maximized. This method is robust in the sense that there need not exist an analytical solution in which three or more curves intersect at a single point. We also avoid the problem of condensing all of the received signal information into one or two parameters (range and variance for example).

2) Rephasing: Once the RF data are captured, the σART algorithm evaluates a metric at every point in a scan-grid. The received data must be shifted in time to remove the delay in the signal that results from the signal traveling distance between a hypothetical mobile transmitter location and the location of reference antenna that captured that signal. This operation, called re-phasing, is performed on each column of $\mathbf{R}$ at every point in the scan-grid in order to cancel the transmission time delays. The time offsets can be precomputed, as we only consider the free-space delay on the direct path of the signal. Since we have a TDOA like system, the global time offsets introduced by traveling through a brick wall, can be represented by a scalar-matrix product with $\mathbf{R}' = e^{-j(\omega\tau_d)}\mathbf{R}$, where τ_d is the additional delay through a dielectric material.

The re-phasing operation itself is just a pre-multiplication with a matrix of time shifts such as in Equation (6). The rephasing process is a unitary transform, and the energy of $\mathbf{R}$ is not altered by re-phasing.

$$||\mathbf{R}|| = ||\Psi(x, y, z)\mathbf{R}|| \tag{7}$$

3) σART metric function: Once the data in $\mathbf{R}$ are re-phased, the σART metric function is computed on the scan-grid. Since the energy of $\mathbf{R}$ is not affected by asynchronous sample clocks, asynchronous mixers, and re-phasing, we can use the first singular value of $\mathbf{R}'$ as a measure of how singular $\mathbf{R}'$ is at each location in the search space. At the correct transmitter location all of the columns of $\mathbf{R}'$ should be linearly dependent, and the entire energy of the matrix contained in σ_1, the first (largest) singular value. In practice, noise and multipath will always put energy into other singular values of $\mathbf{R}'$, but neither of these energy sources is expected to correlate across the columns of $\mathbf{R}'$ to the degree that the direct path signal would. This

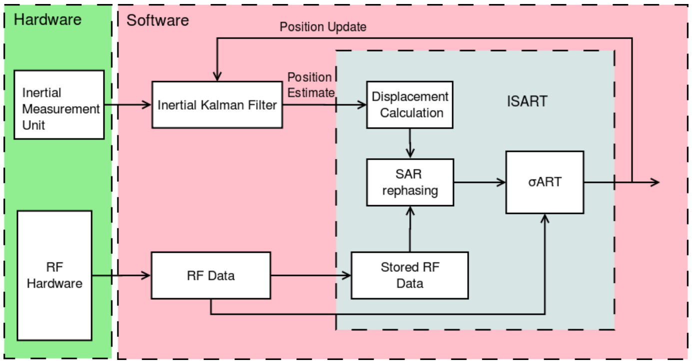

Fig. 1. ISART System Block Diagram

metric function can also provide additional information about the situation such as the location and severity of reflectors. Work has even been done to treat the metric function as a likelihood function for the location of the mobile unit in [10], which expands on analysis conducted by Amendolare [6].

C. ISART

If follows from Equation (7) that a received signal could be re-phased so that it appeared to originate some chosen displacement away from wherever it actually originated. If we use displacement information computed from an Inertial Measurement Unit (IMU) we can perform this additional re-phasing step on RF data captured at previous positions to rectify it to the current position of the user. This re-phasing and combining of RF data effectively produces a synthetic array of antennas extending from the location of each reference antenna, which improves SNR, antenna geometry, and multipath rejection. The block diagram in Figure 1 gives a top-level overview of the algorithm itself. The hardware in the green box can be almost any RF data capture system and IMU. For this paper we used the PPL hardware to capture RF data, and the Analog Devices ADIS16133BMLZ and the Intersense NavChip IMUs to collect inertial measurements. The inertial Kalman Filter is based off of the code provided by OpenShoe.org [4]. The algorithms and data storage are all implemented on a base station computer. Once the stored RF data are re-phased in the Synthetic Aperture Radar (SAR) re-phasing step, the current received data matrix, $\mathbf{R}$ is augmented with the SAR re-phased versions of previous data matrices. For example, on the first RF data capture, there is no displacement $(\Delta x, \Delta y, \Delta z)$, so the augmented matrix, $\underline{\mathbf{R}}$ is just the received data matrix at time zero:

$$\underline{\mathbf{R}}_0 = [\mathbf{R}_0] \tag{8}$$

at the next RF data capture (even if $\Delta x, \Delta y, \Delta z = 0, 0, 0$) the current data matrix is concatenated with a SAR re-phased copy of the data matrix from the previous time step

$$\underline{\mathbf{R}}_1 = [\mathbf{R}_1, \Psi(\Delta x_{0,1}, \Delta y_{0,1}, \Delta z_{0,1})\mathbf{R}_0] \tag{9}$$

in general

$$\underline{\mathbf{R}}_k = [\mathbf{R}_k, \Psi(\Delta x_{k-1,k}, \Delta y_{k-1,k}, \Delta z_{k-1,k})\underline{\mathbf{R}}_{k-1}] \tag{10}$$

Once the matrix is constructed, the σART algorithm is used to determine a position estimate. This estimate is then fed into the INS Kalman filter to correct the error accumulation in the inertial estimates. The error in the ISART position estimate is a zero mean Gaussian random variable, as this is true of the σART position estimate [6][2].

III. EXPERIMENTAL RESULTS

Three field tests have been conducted in order to validate the expected performance gains of the ISART algorithm. This algorithm is designed to improve positioning accuracy in cases where RF data is limited by noise or multipath degradation, lack of antennas, weak signals, or poor antenna geometries. The algorithm also avoids several pitfalls of INS based navigation, specifically: initialization, drift, and the problem of rectifying multiple sensors to a common coordinate frame. The three field tests were conducted in an open auditorium, a wooden house, and a large computer laboratory. These tests are presented in order of perceived difficulty from the standpoint of an RF system, as well as that of an inertial system (path lengths and number of turns increase in each test). It should be noted, however, that the survey error is also increasing from nearly zero up to 20 cm worst case error. In all three cases the RF data were captured with the PPL system, and the inertial data were captured using the Analog Devices ADIS16133BMLZ IMU in the auditorium test, and the Intersense NavChip in the house and lab test. Note that the figures showing INS-only results (Figures 4, 8, and 12) do not have error figures associated with them; this is because it is difficult to compare the INS figure of merit (loop closure error per distance traveled) to the more absolute RMS error criterion on the RF system. To further support this notion, the inertial paths are shown without any initialization, which is exactly how the ISART algorithm employs them.

All of the figures in this section are top down views of the tests being depicted. The circles represent reference antennas, and are generally labeled in a counter-clockwise fashion. The squares are the surveyed truth locations. The actual paths walked all start and end at the same point, so while there are only 17 truth locations surveyed in the auditorium test, there are actually 18 points in the path that was walked.

1) Auditorium: The simplest case that we tested was performed in Alden Memorial Hall on the WPI campus. This is a large, open, indoor space. Both the mobile and reference antennas were located in the same room, so we had line of sight conditions. However, we only use four reference antennas, and two of these were co-located in the x-y plane. The path that we walked was surveyed in the center of the open room, and was relatively short. One feature of this path (see Figure 2) is that it has two 45 degree angles, which required the user to pivot somewhat unnaturally through a 135 degree turn. We found a marked negative correlation between survey simplicity and user comfort. This test used 16 reference antennas, which can be seen in Figure 2, and there were 17 truth points surveyed. In order to walk a closed loop, the first point in the path is also the last point, for a total of

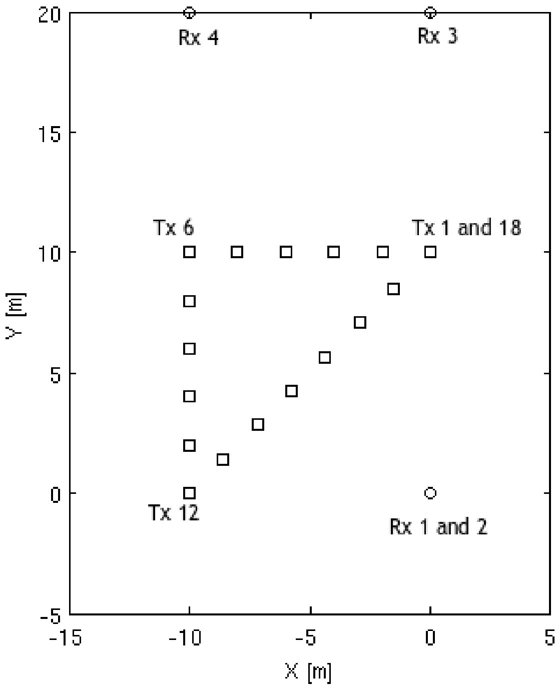

Fig. 2. Alden Hall test configuration

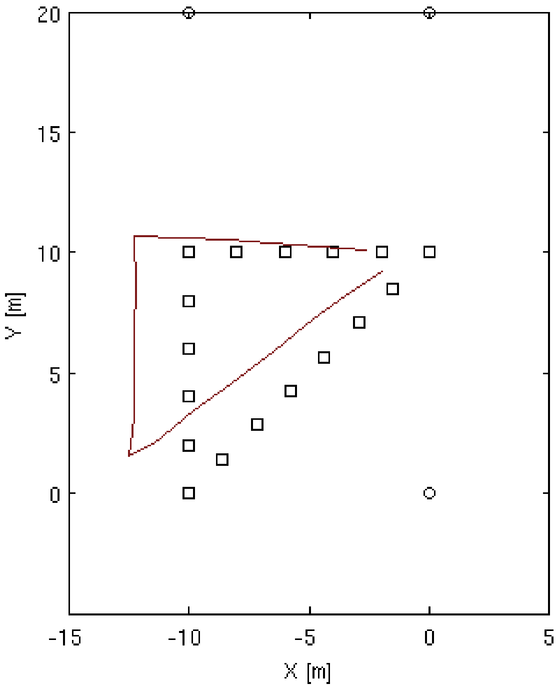

Fig. 4. Alden Hall inertial path

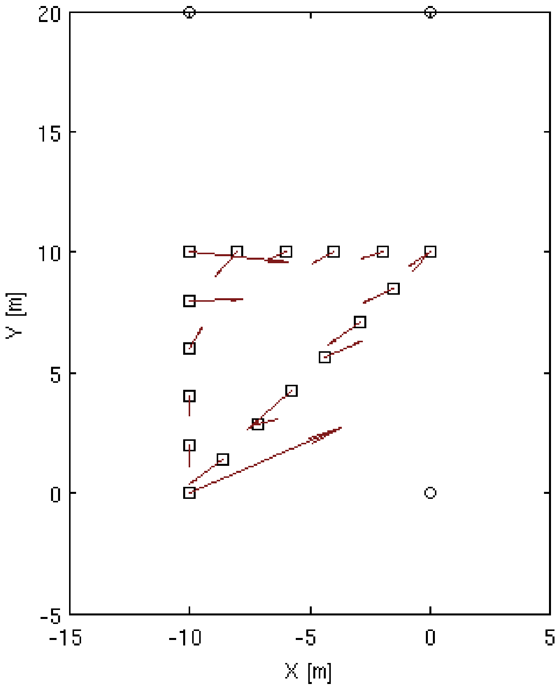

Fig. 3. Alden Hall σART results, RMS error = 2.30 m

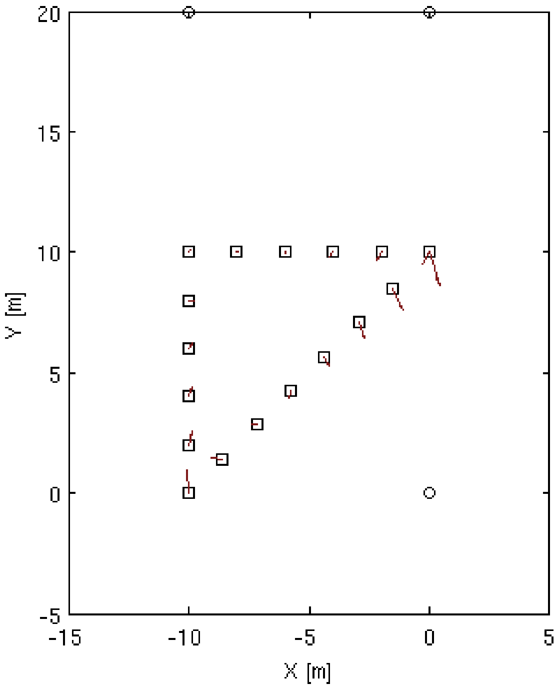

Fig. 5. Alden Hall ISART results, RMS error = 0.58 m

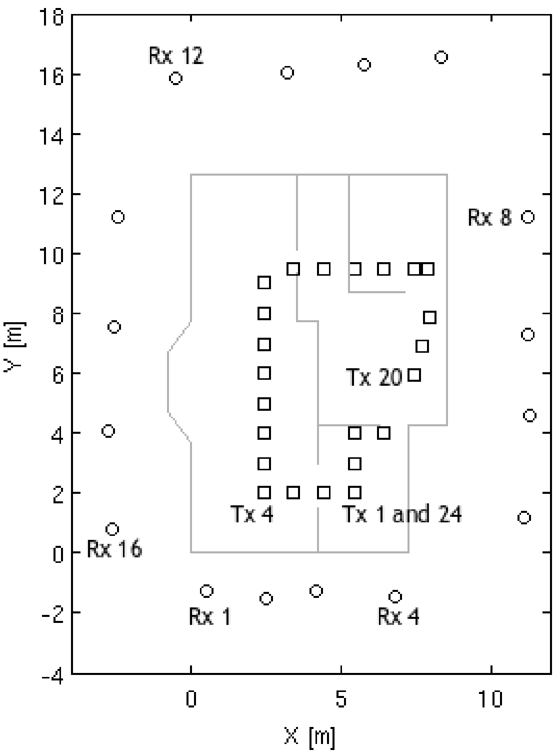

Fig. 6. Wooden House test configuration

18 truth points. Figure 3 shows the results of the RF only σART algorithm in this test, with am RMS error of 2.30 m. The inertial path is shown in Figure 4. Figure 5 shows the ISART result with an RMS error of 0.58 m.

2) *Wooden House:* This test case represents a typical firefighting or law-enforcement scenario. The WPI Campus Religious Center was chosen as it is a house typical of the urban built environment. We deployed our reference antennas on the exterior of the building, and surveyed a closed path that passed through all of the rooms on the first floor. In addition to not having line of sight conditions, this house also has a kitchen and a half, as well as a working fireplace, and several metal cabinets and desks. The length of the path was longer than that of the previous test and required the user to pivot a maximum of 90 degrees. This test used four reference antennas, which can be seen in Figure 6, and there were 23 truth points surveyed. In order to walk a closed loop, the first point in the path is also the last point, for a total of 18 truth points. Figure 7 shows the results of the RF only σART algorithm in this test, with am RMS error of 2.30 m. The inertial path is shown in Figure 8; Figure 9 shows the ISART result with an RMS error of 0.77 m. Although this was a more difficult test scenario than the auditorium test, the σART results were comparable; this is largely due to the fact that the number of antennas and their geometric configuration was far superior to that of the previous test. This is an important point because the ISART algorithm effectively multiplies the number of reference antennas, and

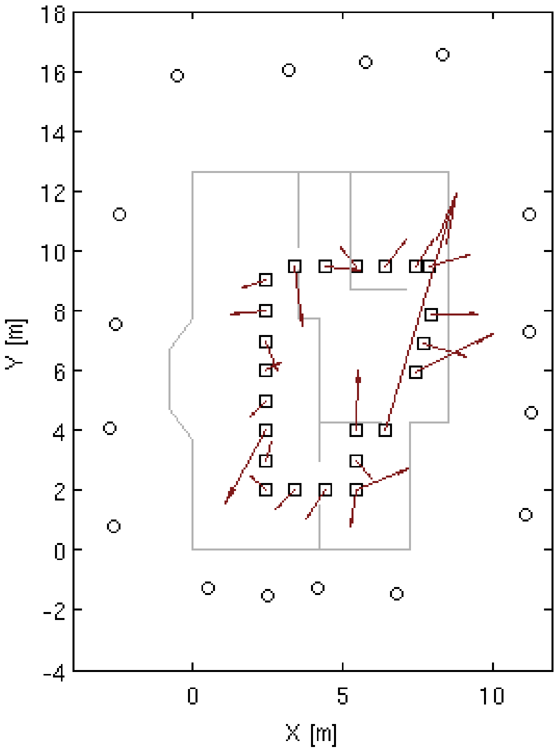

Fig. 7. Wooden House σART results, RMS error = 2.20 m

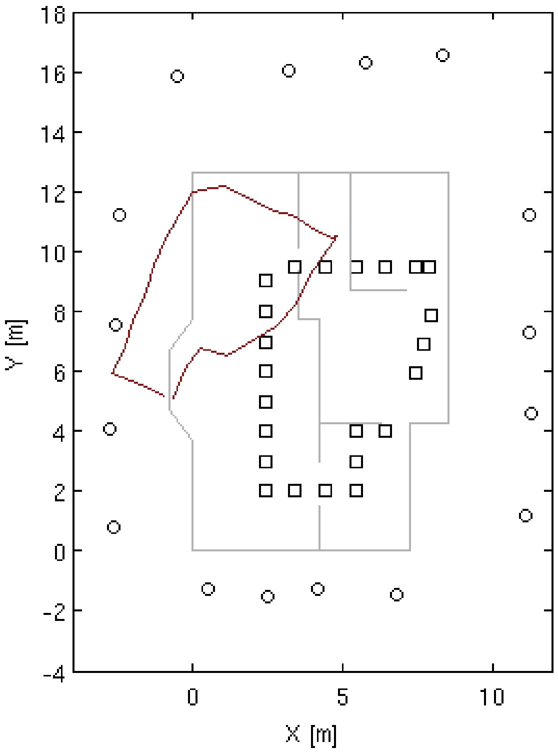

Fig. 8. Wooden House inertial path

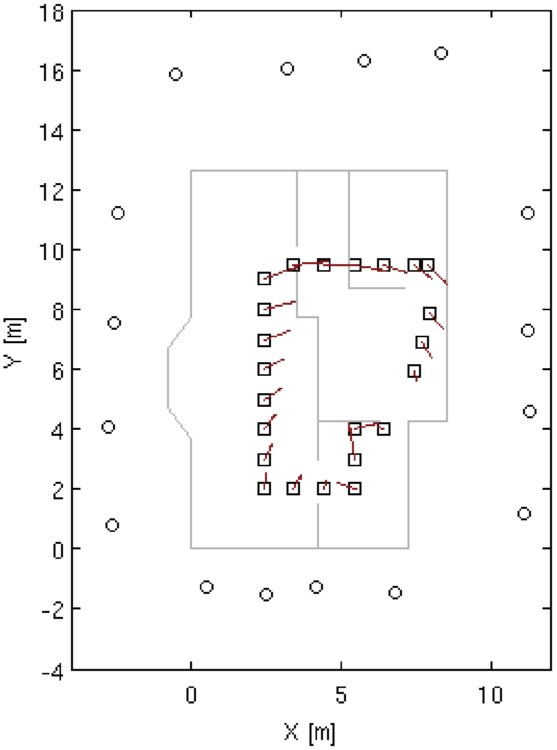

Fig. 9. Wooden House ISART results, RMS error = 0.77 m

should improve their geometric configuration according to the path of the mobile unit.

3) Computer Lab: This test case represents a more difficult firefighting or law-enforcement scenario than the previous test. This room is more typical of a small commercial or industrial building with metal construction, metal furniture, and of particular interest, an aluminum white-board covering most of the leftmost wall (between the reference antennas and all of the truth points) in Figures 10-13. We deployed our reference antennas on the exterior of the lab, and in order to make the inertial path as natural as possible for the user, we had the user walk a path through the room, which we marked and post-surveyed. This was by far the easiest path to walk and the hardest to survey. Because of this, we accumulated up to 20 cm of survey error at the points farthest from the start of the path. This test used 16 reference antennas, which can be seen in Figure 10, and there were 23 truth points surveyed. In order to walk a closed loop, the first point in the path is also the last point, for a total of 18 truth points. Figure 11 shows the results of the RF only σART algorithm in this test, with am RMS error of 2.82 m. The inertial path is shown in Figure 12. Figure 13 shows the ISART result with an RMS error of 1.77 m. Unfortunately we were unable to achieve a sub-meter accurate positioning solution in this test. The most likely cause for this is the fact that reference antennas 1-4 (see Figure 10) were blocked by a white-board. In addition to being a strong reflector, due to metallic backing material, this board is blocking the direct path signal to these antennas completely.

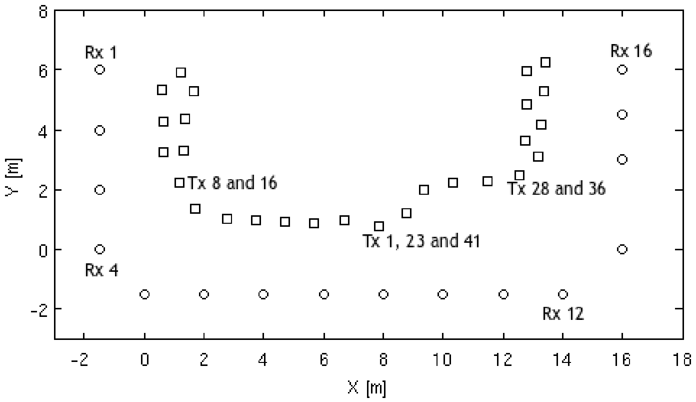

Fig. 10. Computer Lab test configuration

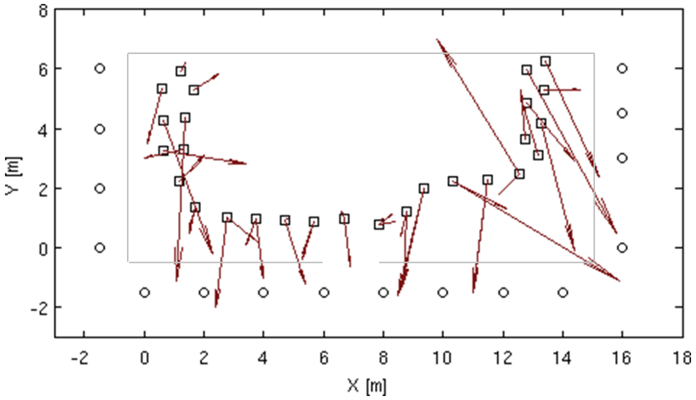

Fig. 11. Computer Lab σART results, RMS error = 2.82 m

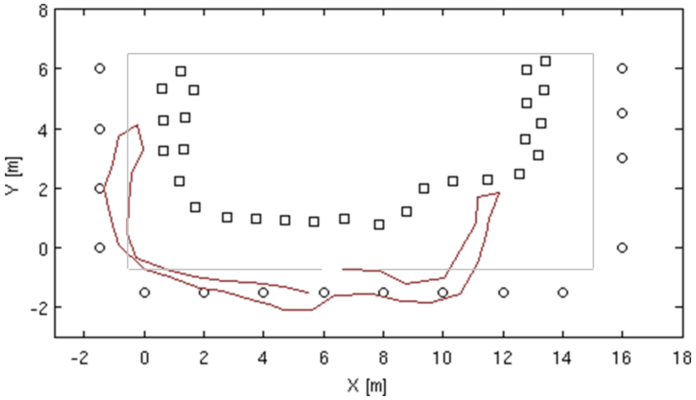

Fig. 12. Computer Lab inertial path

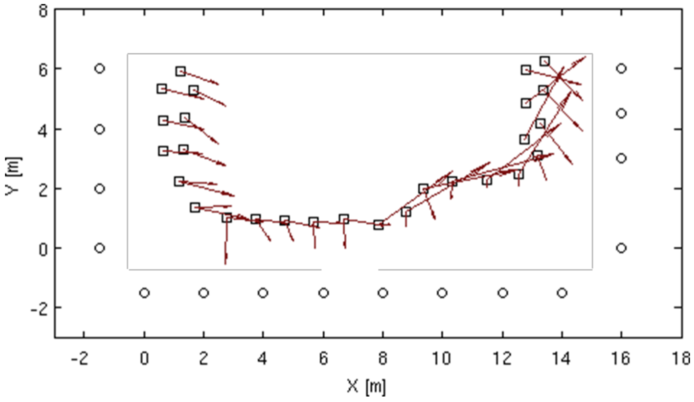

Fig. 13. Computer Lab ISART results, RMS error = 1.77 m

The result is that the data from these antennas is based off of the strongest reflection, which by geometric examination must be longer than the direct path. This consistent path-lengthening shows up as a visible bias of the error vectors in Figure 13 to the right hand side of the room. Situations like this one present a problem for any RF based system.

IV. CONCLUSIONS AND FUTURE WORK

This paper presented a new framework for INS-RF sensor fusion that differs significantly from other fusion techniques. The fusion of RF data at the signal level from inertial measurements, as well as the use of array processing techniques can be applied to almost any INS and RF data sources. The stationary nature of the RF data limits the error growth of the INS, as well as providing initialization and coordinate frame rectification. The short-time INS displacement information is used to combine spatially diverse RF data samples, and aid in the mitigation of multipath interference. This algorithm differs significantly from the state of the art in INS/RF fusion techniques, and is designed specifically to deal with the problem of multipath.

Three experiments were conducted to validate the performance gains from the ISART algorithm over the existing σART algorithm. The federated INS/RF system, using ISART, is superior to our existing RF-only σART algorithm (see the error performance summarized in Table I), and solves the problems of initialization and coordinate frame rectification better than an INS-only system could ever do. There are still scenarios in which the ISART algorithm can fail, and one of these was encountered in our third field test. The system was consistently biased in estimating the positions of the mobile unit in a room that contained a large uniform metal reflector (white board). In this type of scenario the effects of such a reflector would likely diminish if the mobile unit were to move sufficiently far from the reflector, or if the user were walking around (orbiting) the reflector. The latter case would be especially helpful in our field test, as it would relieve the left-hand antenna array from it's blocked direct-path condition. It is also important to note that two users attempting to navigate to one another would not suffer from this bias in the estimate of their relative positions. In all three cases, the ISART algorithm performed significantly better than the σART algorithm operating on the same data. TOA-like timing information may also enable better spatial filtering to deal with large reflectors, or blocked direct-path conditions [11]. In addition to investigating larger scenarios, and incorporating advanced synchronization schemes into the ISART algorithm there needs to be a real-time implementation of ISART. Currently the code runs in Matlab and can only work in a post-processed operating mode. Fortunately the ISART algorithm is highly parallelizable. For example, the scan grid for the wooden house test contains 2304 points, all of which can be processed by separate threads.

TABLE I
ERROR SUMMARY

Test	RMS error σART	RMS error ISART
1	2.30 m	0.58 m
2	2.20 m	0.77 m
3	2.82 m	1.77 m

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REFERENCES

[1] A. Cavanaugh, M. Lowe, D. Cyganski, and R. J. Duckworth, "WPI Precision Personnel Locator System: Inverse Synthetic Array Reconciliation Tomography," in *IEEE/ION Position Location and Navigation Symposium (PLANS) Conference*, Apr. 2012.

[2] A. Cavanaugh, M. Lowe, D. Cyganski, and R. J. Duckworth, " WPI Precision Personnel Location System: Rapid Deployment Antenna System and Supporting Algorithms for 3D Precision Location," in *ION International Technical Meeting*, Jan. 2010.

[3] R. J. Duckworth, D. Cyganski, S. Makarov, W. Michalson, J. Orr, V. Amendolare, J. Coyne, H. Daempfling, D. Hubelbank, H. Parikh, and B. Woodacre, "WPI precision personnel locator system: Evaluation by first responders," in *Proceedings of ION GNSS*, (Fort Worth, Texas), September 2007.

[4] J.-O. Nilsson, I. Skog, P. Handel, and K. Hari, "Foot-mounted INS for Everybody An Open-source Embedded Implementation," in *IEEE/ION Position Location and Navigation Symposium (PLANS) Conference*, Apr. 2012.

[5] E. Foxlin, "Pedestrian Tracking with Shoe-Mounted Inertial Sensors," *IEEE Computer Graphics and Applications*, Nov. 2005.

[6] V. T. Amendolare, *Transactional Array Reconcilliation Tomography for Precision Indoor Location*. PhD thesis, Worcester Polytechnic Institute, 2010.

[7] S. Boyd, "Multitone Signals with Low Crest Factor," *IEEE Transactions on Circuits and Systems*, vol. CAS-33, pp. 313–319, Oct. 1986.

[8] D. J. Newman, "An L1 Extremal Problem for Polynomials," *Procedings of the AMS*, vol. 16, pp. 1287–1290, Dec. 1965.

[9] D. E. Breen, "Characterization of Multi-carrier Locator Performance," Master's thesis, Worcester Polytechnic Institute, 2004.

[10] A. Cavanaugh, "Bayesian Information Fusion for Precision Indoor Location," Master's thesis, Worcester Polytechnic Institute, 2010.

[11] V. Amendolare, A. Cavanaugh, D. Cyganski, and R. J. Duckworth, "WPI Precision Personnel Locator System: New RF Location Algorithms for Improved Precision Location in High Multipath Environments," in *Proceedings of the Institute of Navigation Joint Navigation Conference*, (Orlando, Florida), June 2010.